

the ionising potential for negative corpuscles, the positive ions being derived from heated salts.

4. In a new experimental arrangement, measurements of the ionising potential for negative corpuscles were repeated, and the values, 11 volts for hydrogen and 20 volts for helium, were obtained, which confirm the results of Franck and Hertz.

5. Observations were made on positive ions emitted by heated sodium phosphate in hydrogen, oxygen, nitrogen, and helium.

Further experiments are in progress. When this research had been conducted to the point described above, a paper by E. v. Bahr and J. Franck appeared in the 'Verhandlungen der Deutschen Physikalischen Gesellschaft,' January, 1914, which contains results which are in agreement with those of the experiments described in § 2 of the present investigation.

The author 'gladly takes this opportunity of thanking Prof. Sir J. J. Thomson for his kind permission to carry on this research in the Cavendish Laboratory, and for the interest he has taken in the investigation.

The Determination of Fatigue Limits under Alternating Stress Conditions.

By C. E. STROMEYER.

(Communicated by Prof. W. E. Dalby, F.R.S. Received March 26,—
Read May 21, 1914.)

In the year 1867 A. Wöhler, locomotive superintendent of a railway company in Berlin, exhibited at the Paris Exhibition the results of some experiments on the endurance of metals, and was thereupon engaged by the Prussian Government to carry out the more exhaustive enquiry into this subject with which his name is always associated. The results of his labours were published in 1871, and were highly appreciated, but few additional experiments were made until the subject was again taken up successively by Sir Benjamin Baker, Reynolds and Smith, Rogers, Stanton and Bairstow, Eden, Rose and Cunningham, and Prof. Hopkinson. All these experiments are confined either to fatigue bending or to push and pull tests, using only steel or iron, whereas the present ones include a large number of torsion fatigue tests on various metals.

Until comparatively recently there was no satisfactory standard of comparison for fatigue tests, the determination of the asymptote or limiting fatigue stress for an infinite number of revolutions from a few irregular test results leading to very uncertain conclusions, so much so that by some it was considered very doubtful whether there were any real fatigue limits, while others adopted as standards of comparison the fatigue stresses which would cause fractures at the millionth repetition. The first problem which had to be investigated was therefore to ascertain the relationship between the intensities of fatigue stresses and the numbers of repetitions of these stresses which would cause fracture; and, should this relationship be found to indicate the existence of a limiting stress for an infinite number of revolutions, or more briefly of a fatigue limit, then the next step would have to be its exact determination.

A discussion of Wöhler's tests, which has often been attempted, did not seem to be likely to lead to any definite results, for he was evidently under the impression that the quality of steel with which he was being supplied by several steel makers did not vary during a twelvemonth, and instead of making a series of tests with small samples cut from a single axle, he seems to have made his test-pieces so large that a separate axle had to be used for each one, although the first may have been supplied in January and the other in December of a single year. Since his days it has become known that, even in a single ingot, comparatively large differences exist as regards chemical composition and mechanical properties, according as to whether the samples are taken from near the surfaces of the ingots or from the tops or bottoms of their cores. Therefore, in order to reduce to a minimum the chances of obtaining irregular fatigue results, the author made use of very short test-pieces, so that the distance from sample to sample did not exceed 3 inches and for the bending tests was as small as 1 inch. The shape of the samples used for bending fatigue tests is shown in fig. 1, each test-piece consisting of



FIG. 1.

seven waists and cones. The end cone of each sample was firmly inserted into the socket of a horizontal revolving spindle (fig. 2), and to the other end of the test-piece was screwed a hardened steel rod from which a weight was suspended. After revolving the spindle for a certain number of times the first (left hand) waist would break and the second cone would be inserted in the socket, and so on until all the waists were broken. The load on the end of the bar was not altered, so that the bending moment on any one waist

would be the same throughout the entire duration of the test. With this form of test-piece the six successive fractures would be separated from each other by only 1 inch.

With the permission of Prof. Dalby, his assistant, Mr. (now Professor) A. J. Margetson, carried out a series of bending fatigue tests during 1907

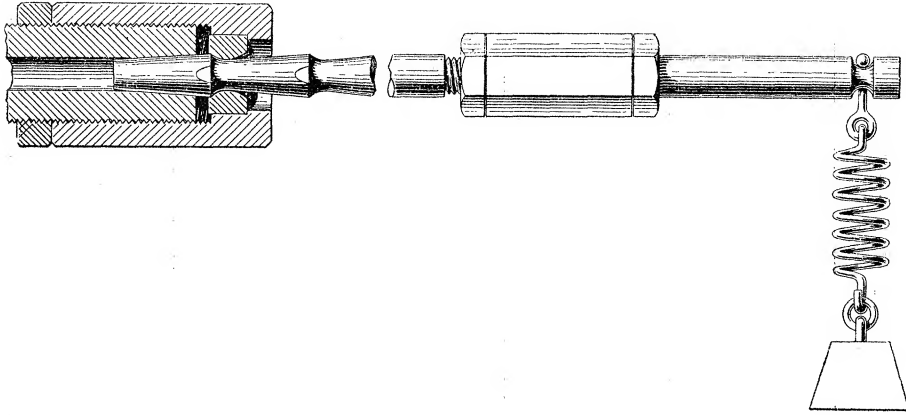


FIG. 2.

and 1908 on about 30 different qualities of steel, of which a large number had been cut from plates which had failed in practice. His results have been carefully analysed and were found to comply with the empirical condition

$$\pm S_n = Fl + C (10^6/N)^{1/4},$$

where Fl is the extrapolated fatigue limit, C is a coefficient affecting the endurance under severer stresses, and $\pm S_n$ is the nominal fatigue stress which will cause rupture if repeated N times; N being the total number of stress cycles or revolutions up to fracture. The nominal fatigue stresses $\pm S_n$ are calculated from the alternate bending moments as if the material were perfectly elastic up to these stresses, whereas, on account of the plasticity of steel under severe stresses, the actual fatigue stresses are necessarily lower than these nominal ones, except for vanishing differences of $S_n - Fl$. The correction for reducing S_n to S will therefore most likely not affect Fl , the extrapolated fatigue limit, which is the main object of the present enquiry, but it will certainly affect the coefficient C and, possibly, also the index $1/4$.

The results of these bending tests have been recorded diagrammatically in figs. 3 and 4, the vertical scales of the diagrams representing the nominal bending fatigue stresses $\pm S_n$, and the horizontal ones representing the expression $(10^6/N)^{1/4}$. Seeing that the distance which separates the first

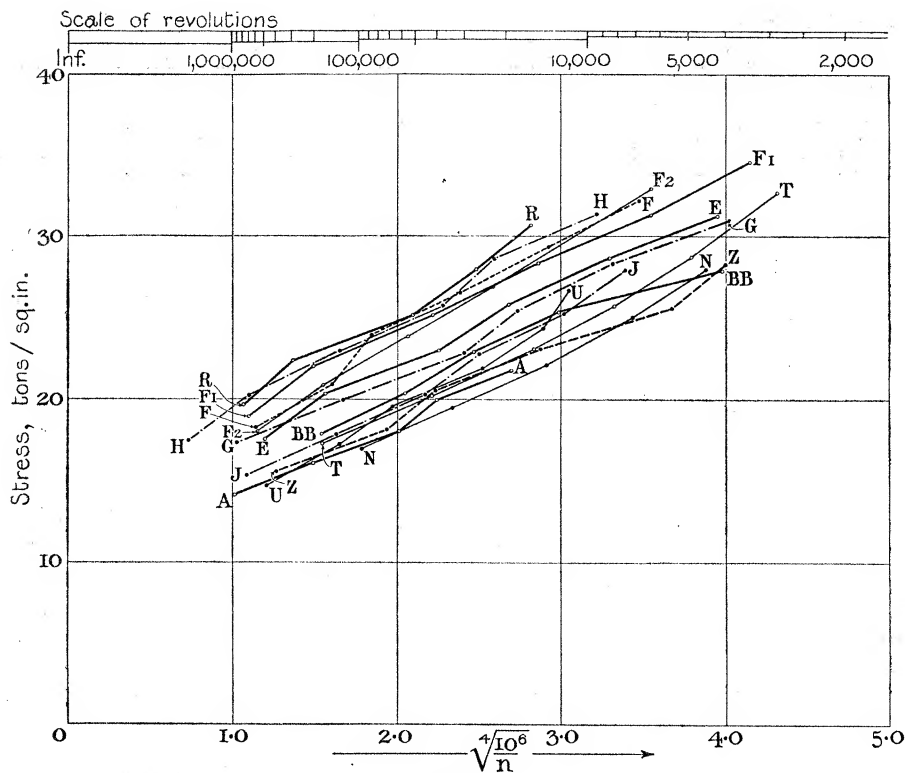


FIG. 3.

fractured waist from the last (see fig. 1) is only 6 inches, it was hoped that, within this length, no change of endurance qualities would occur, and that the intersections of the prolongations of the plotted test results with the zero ordinate might be looked upon as being the true fatigue limits, but, judging by some of Messrs. Eden, Rose, and Cunningham's test results, very marked changes of endurance qualities are met with even in short lengths of rods, and with the form of test-piece adopted in the present trial, these local changes would introduce systematic errors, either altering the slopes of the lines in figs. 3 and 4 or curving them, and these influences would indirectly affect the extrapolated fatigue limits F_1 . Partly for this reason, partly because of the exceptional chemical compositions of the steels marked D and F, these were subsequently tested in duplicate and the results are as follows:—

Sample D..... $F_1 = 10.92$ and 12.79 ; $C = 3.78$ and 3.74 .

Sample F..... $F_1 = 11.49$, 11.75 and 14.33 ; $C = 5.98$, 6.02 , 4.85 .

These agreements and disagreements could easily be explained by assuming certain slight local differences of endurance qualities, but as they indicate

was recorded by counters. The testing machine is illustrated in fig. 5 (elevation) and fig. 6 (plan).

See S is the central crankshaft with a heavy flywheel F, which gives steadiness of rotary motion. It is driven at about 600 revolutions per minute by a belt from the motor M. The throw or double radius of each

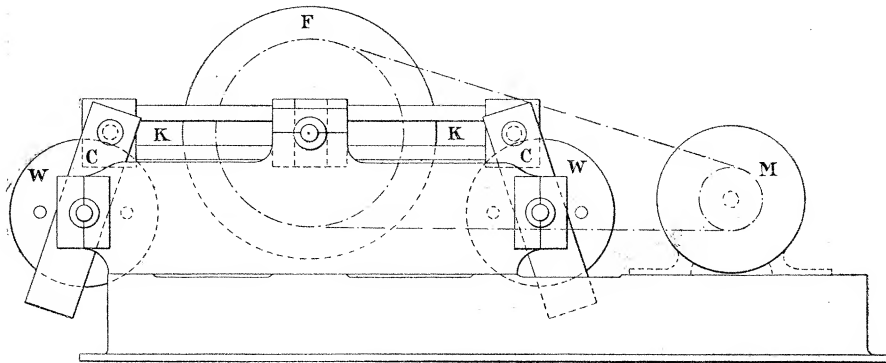


FIG. 5.—Elevation.

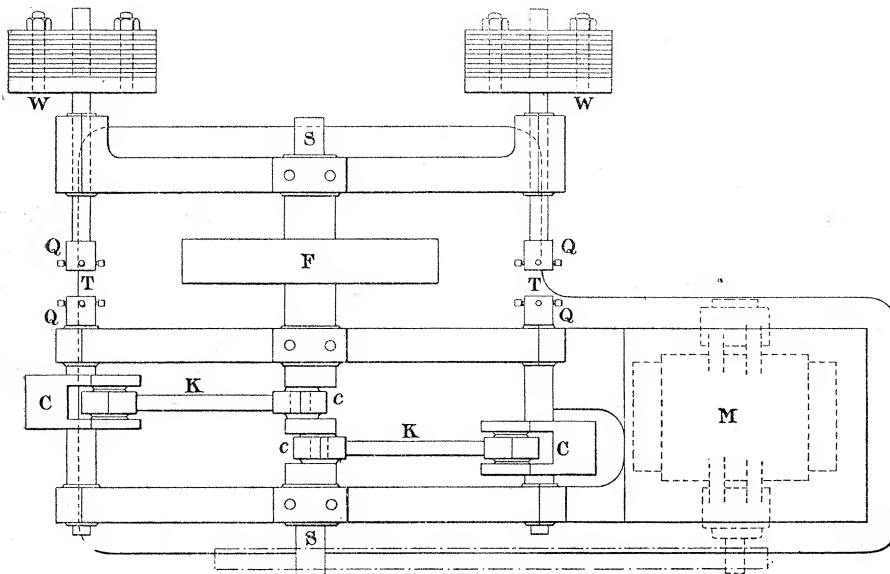


FIG. 6.—Plan.

crank is 1 inch. Two connecting rods KK join the rotating cranks *cc* to the balanced rocking cranks CC, whose radii of action are 4 inches. The shafts of these cranks end in clutches QQ, and the spindles of the two flywheels WW have similar clutches, into which the ends of the test-pieces TT can be secured. The flywheels WW are built up of a number of circular

discs, whose moments of inertia have been accurately determined. The cranks CC are, as will be seen, inclined at angles of about $22\frac{1}{2}^\circ$, which angles can be slightly altered by adjusting the lengths of the connecting rods KK. This inclination of the cranks CC is an essential feature of the machine, for without it the torsion moments would not be equal at both ends of the stroke, and the test-pieces would acquire increasing twists in one direction while the test is in progress.

The shapes of the test-pieces held by the clutches QQ, and the attachments for the calorimetric measurements are shown in fig. 7. For the latter purpose the test-piece T is surrounded by a thick indiarubber sleeve, shown in section, to which an inlet and an outlet pipe are cemented; sensitive thermometers t_1 and t_2 being placed in each pipe. A steady stream of water enters at I, its temperature being read off at t_1 ; it travels along the test-piece T, taking up any heat which may be generated in it, and passes out at O, the discharge temperature being recorded by t_2 . When much heat is generated in a test-piece a small portion escapes through the clutches QQ (fig. 6), because these would be relatively cold, and if the clutches are warmer than the test-piece heat will travel into it. In order to determine

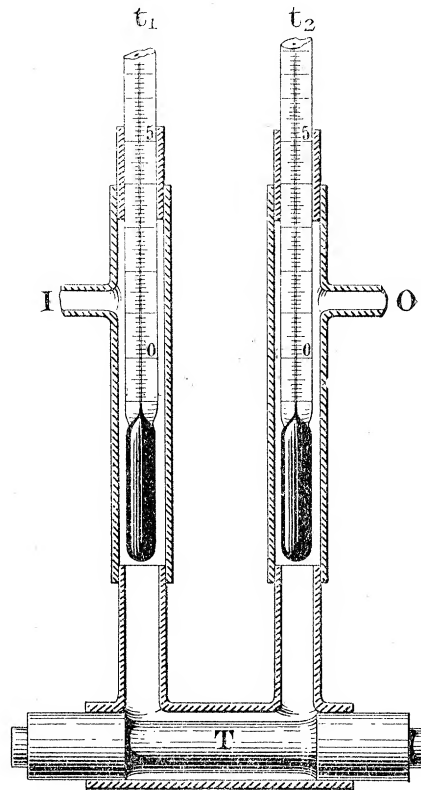


FIG. 7.

the amount of this conducted heat, holes were drilled into the bearings near QQ and thermometers inserted, whereby the flow of heat from or to T was determined in terms of the difference of temperature between that of the test-piece and the two bearings. The flow of the cooling water was maintained at a very steady rate by a constant head of pressure, and was frequently checked. It could be reduced to 2 c.c. per minute, and as the temperature could be read to 0.01°C. , the calorimetric attachment would be capable of just indicating 0.02 small calories per minute, or 1.43 gm.-cm. of work per revolution or stress cycle, of which there were about 600 per minute. This plastic work is represented in fig. 8 by the area of the

loop $abcd$, OF being the maximum twisting moment and Fb the twisting angle of the test-piece. For test-pieces of the dimensions used in these experiments the elastic work, if added together for each quarter cycle, or twice the product $OF \times Fb$, was from 1000 to 3000 times larger than the area $abcd$.

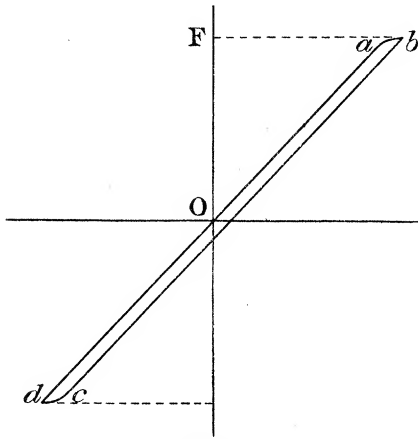


FIG. 8.

In the present experiments the accuracy was reduced to about 1 per cent. by increasing the flow of water to about 30 c.c. and by disregarding differences of temperatures of less than 0.02°C .

Before proceeding with the calorimetric determination of fatigue limits, it was deemed desirable to subject all the samples, of which the bending fatigue tests are recorded in diagrams 3

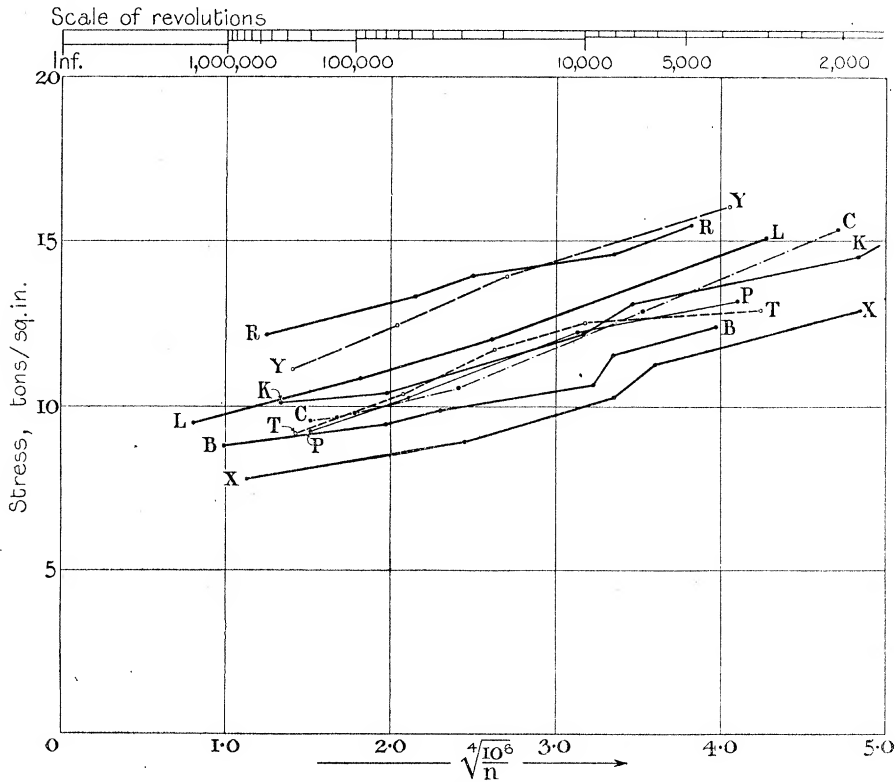


FIG. 9.

and 4, to torsion fatigue tests, using waisted samples of the very shortest dimensions ($1\frac{1}{2}$ inch over all). The experiments were mostly carried out by my assistant, Mr. H. A. Jones, B.Sc., and the results are recorded diagrammatically in figs. 9 and 10, which are arranged on the same plan as diagrams 3

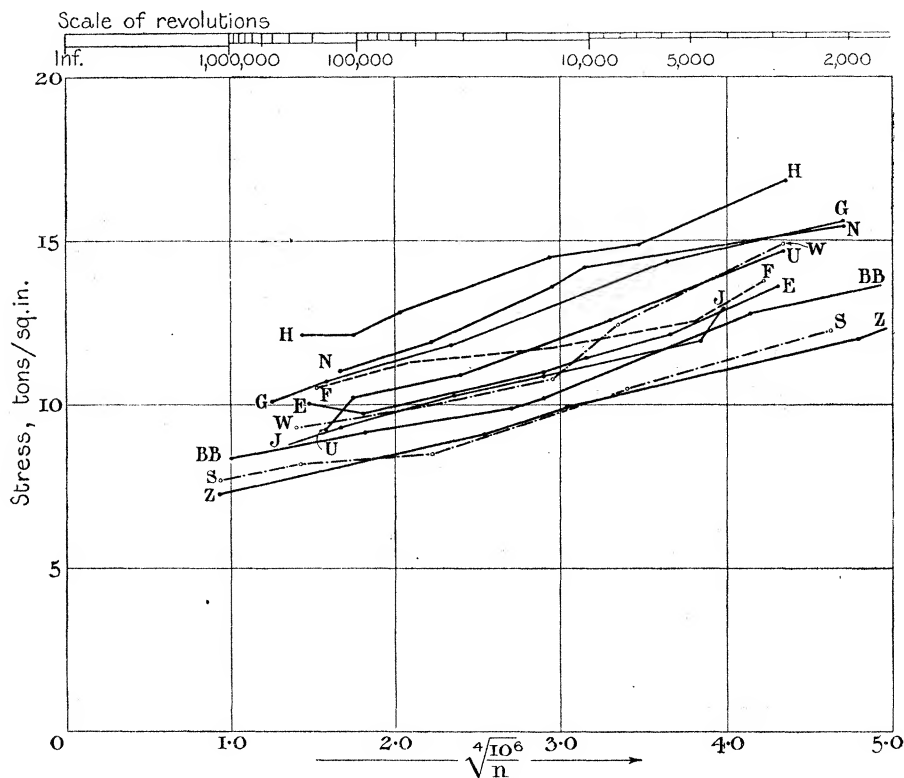


FIG. 10.

and 4, except that the vertical scales (stresses) are twice as large as the previous ones. As will be seen these plotted torsion fatigue test results lie on practically straight lines, which is an indication that a definite fatigue limit also exists for torsion fatigue stresses, and should be sought for where the prolongations of the several lines cut the zero ordinate.

It may here be mentioned that the extrapolated fatigue limits for torsion (shear) average two-thirds of those for bending, and that the coefficients C for torsion average 0.3 of those for bending, but individual ratios differ from the means up to about ± 25 per cent. All the previous tests had been made on bars cut from plates, but as some of these plates showed laminar segregations, which may have affected the results, all the subsequent tests have been made with samples cut from rolled bars.

The calorimetrically determined fatigue limit is that minimum alternating stress which will just generate heat in the test-piece. This generation of heat is almost certainly an indication that the material of the test-piece is being fatigued by being alternately stressed beyond its elastic limit, and that if the process could be continued an infinite number of times the test-piece would fracture. But in the previously mentioned empirical formula $\pm S_n = F1 + C(10^6/N)^{1/4}$ the fatigue limit $F1$ is found by extrapolating those alternating stresses which result in actual fractures, which limit is therefore that stress which will cause fracture if repeated an infinite number of times, and if this empirical formula should be applicable to an infinite number (N) of stress cycles, then the fatigue limit as found calorimetrically in a test of short duration should agree reasonably well with it. In order to clear up this point five bars were tested—LA, LB, LD, LE, LF—and the results recorded in Table I and in diagram 11.

As will be seen, the agreement between the two methods of testing is a remarkably close one, the stresses, as estimated from the observed number of

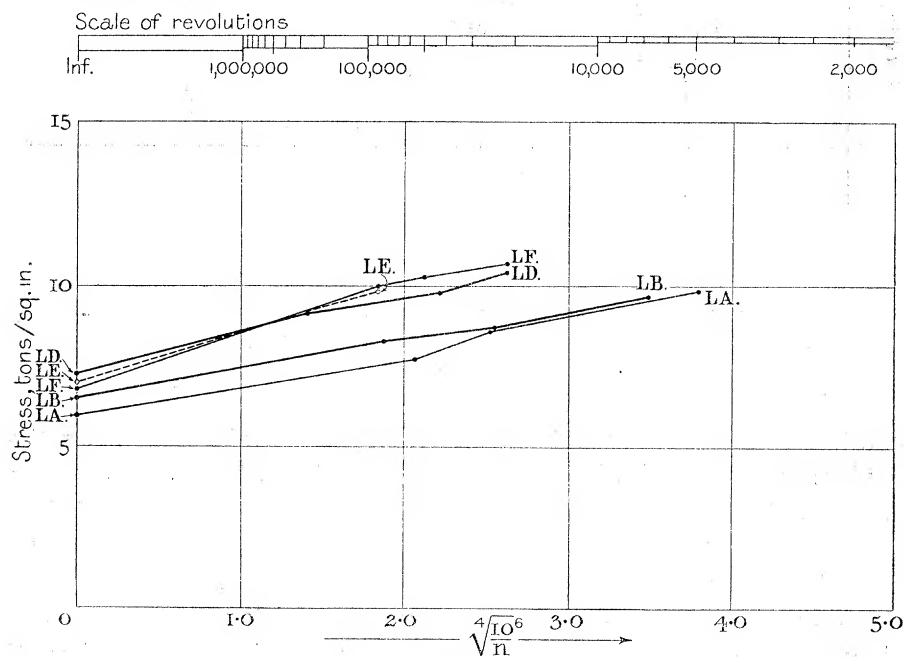


FIG. 11.

revolutions, agreeing with the applied stresses more closely than might have been expected from the irregularities of the curves in Diagrams 3, 4, 9, 10. The worst case is LF, for which the differences are -0.52 and $+0.60$ ton per square inch, but these could be nearly eliminated if in the formula

Table I.—Comparative Torsion Fatigue Tests.

Sample mark.	Fl, calorimetric.	± S _n , fatigue stresses.	N, revolutions to fracture.	(10 ⁶ : N ^{1/2} .)	Fatigue stresses.	
	Tons per square inch.	Estimated.			Differences.	
		Tons per square inch.				
Basic Open Hearth Mild Steel Bar.						
LA 21	6.15	9.85	4,840	3.790	9.67	+0.18
LA 19	6.00	8.55	23,240	2.560	8.47	+0.08
LA 20	5.75	7.70	56,400	2.5	7.97	-0.27
Mean	5.97	(Mean coefficient, C = 0.977)				
Basic Open Hearth Mild Steel Bar.						
LB 5	6.60	9.70	6,870	3.478	9.57	+0.13
LB 6	6.50	8.62	23,627	2.550	8.77	-0.15
LB 7	6.50	8.20	82,100	1.867	8.16	+0.04
Mean	6.53	(Mean coefficient, C = 0.701)				
Basic Open Hearth Mild Steel Bar.						
LD 6	7.30	10.40	21,440	2.615	10.43	-0.03
LD 5	7.10	9.80	37,515	2.272	10.01	-0.21
LD 7	7.25	9.18	242,000	1.426	8.97	+0.21
Mean	7.22	(Mean coefficient, C = 1.225)				
Basic Bessemer Mild Steel Bar.						
LE 19	6.96	10.04	91,035	1.820	9.94	+0.10
LE 17	7.00	9.76	99,900	1.777	9.87	-0.11
LE 18	6.95	9.72	123,800	1.684	9.71	+0.01
Mean	6.97	(Mean coefficient, C = 1.630)				
Basic Bessemer Mild Steel Bar.						
LF 18	6.88	10.70	21,200	2.620	11.22	-0.52
LF 19	6.94	10.30	49,300	2.124	10.39	-0.09
LF 17	6.67	9.92	206,600	1.484	9.32	+0.60
Mean	6.83	(Mean coefficient, C = 1.675)				

$\pm S_n = Fl + C(10^6/N)^{1/4}$ the index $1/4$ were replaced by a smaller fraction. It is, however, probable that these discrepancies are due to irregularities in the material. Probably the most important result revealed by these experiments is the definiteness of the calorimetrically determined fatigue limits, the maximum differences from any of the mean values in Table I not exceeding 0.22 ton per square inch. This result will appear all the more satisfactory when it is remembered that the static elastic limits for mild steel are such very uncertain quantities that engineers refuse to recognise them as being of any practical value, and prefer to judge of the qualities of mild steels by their ultimate tenacities, although these may vary by more than ± 5 per cent. in a single plate. This definiteness of the fatigue limit was confirmed in a most gratifying manner by tests of 19 samples cut from a single crankshaft. Of these, three were cut from the scrap end of the shaft, probably near its surface, and the fatigue limits were found to be 11.15, 11.40, and 11.25 tons per square inch (11.27 mean). Eight other samples were cut longitudinally from the centres of the four crank webs close to the crank pins. Their fatigue limits were 12.15, 12.05, 12.10, 12.10, 12.15, 12.00, 12.15, 12.05 tons per square inch (12.09 mean). Eight other samples were cut from the same positions, but across the grain; the results were 12.15, 11.30, 11.60, 11.30, 11.60, 11.35, 12.00, 11.30. The maximum difference between the mean and individual results were respectively 0.13, 0.09, and 0.57 ton per square inch, which is a most satisfactory result.

The calorimetric method of determining fatigue limits is not only simple and expeditious, but, as the above tests seem to indicate, also reliable, and may possibly help to throw new light on several as yet unsolved problems of mechanics, more especially if, as can easily be done by fracturing the sample, the coefficient C is also determined, which gives some indication as to the power of the material to withstand a few severe alternating stresses. The rate of heat evolution during the fatiguing process has as yet not been correlated to any mechanical properties of the materials tested, but there seems to be a rough relation to the fatigue stress. Thus, for three mild steels, two cast steels, and aluminium, the ratio of the heat H evolved during the early portions of the fatiguing processes to the square of the ratio of the excess stress over the fatigue limit $(S_n - Fl)$ to the fatigue limit Fl , viz., $H \div [Fl/(S_n - Fl)]^2$, is reasonably constant for each material, and the extreme mean values of C for such different materials as steel and aluminium are comprised within the limits of 329 and 548 (see Table II).

Table II.—Fatigue Stresses and Rate of Evolution of Heat.

Sample.	ϕ . Ratio of excess of stress over fatigue limit to fatigue limit. ($S_n - FI$)/ FI .	H. Heat evolved per minute per cubic inch of sample.			Ratios.	
		Initial stage.		Mean.	H_2/ϕ^2 .	H_3/ϕ^2 .
		H_1 . From	H_2 . To	H_3 .		
		inch-pounds.				
Three Samples of Mild Steel.						
LA 20	0.323	20	60	66	575	633
LA 19	0.427	15	95	95	522	522
LA 21	0.620	200	210	260	547	677
Mean	—	—	—	—	548	611
LB 7	0.260	7	30	56	444	830
LB 6	0.325	0	50	118	474	1120
LB 5	0.475	87	110	243	488	1080
Mean	—	—	—	—	469	1010
LD 7	0.267	0	17	43	239	604
LD 5	0.374	0	72	125	516	898
LD 6	0.430	34	55	181	298	980
Mean	—	—	—	—	351	827
Two Samples of Cast Steel.						
AA 3	0.430	0	21	33	114	179
AA 1	0.607	70	160	131	434	355
AA 2	0.795	105	220	273	349	432
Mean (omitting AA 3)	—	—	—	—	391	393
AB 1	0.441	0	52	57	267	293
AB 2	0.573	120	145	134	442	409
AB 3	0.744	65	165	179	299	324
Mean	—	—	—	—	336	335

Table II—*continued.*

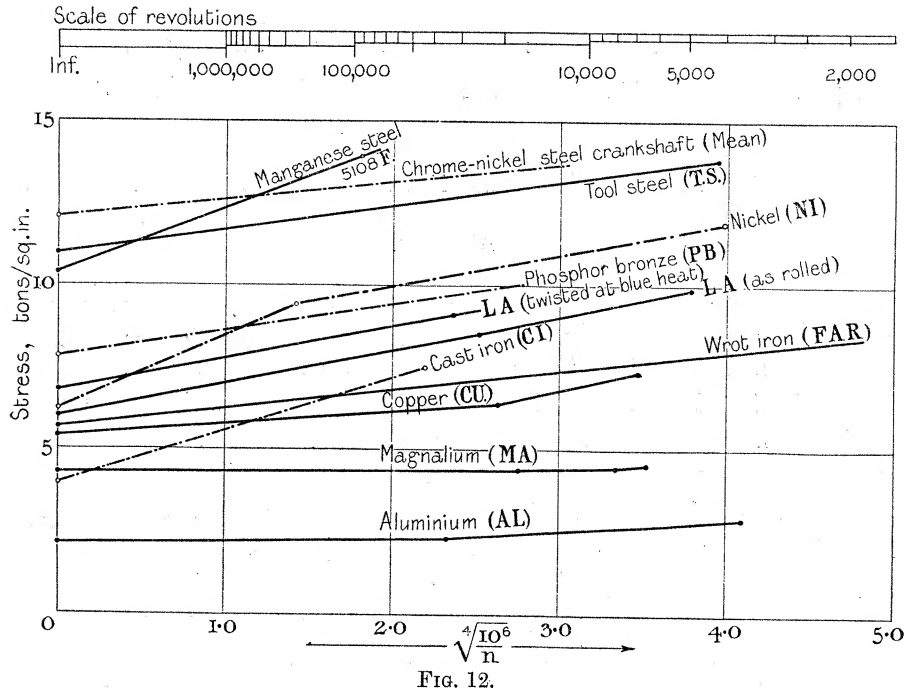
Sample.	ϕ . Ratio of excess of stress over fatigue limit to fatigue limit. ($S_n - F_l$)/ F_l .	H. Heat evolved per minute per cubic inch of sample.			Ratios.	
		Initial stage.		Mean.	H_2/ϕ^2 .	H_3/ϕ^2 .
		H_1 . From	H_2 . To	H_3 .		
		inch-pounds.				
One Sample of Aluminium.						
AL 3	0.086	1.4	2.7	2.1	367	284
AL 1	0.154	3.3	5.7	4.9	241	207
AL 2	0.313	9.5	37.0	23.6	378	242
Mean	—	—	—	—	329	244

Having established the existence of definite fatigue limits for mild steel, additional torsion fatigue tests were made with other materials, the results being recorded in Table III and diagrammatically in fig. 12. The latter shows very clearly that the endurance qualities of the tested metals can be expressed by the same formula as that which was found applicable to steel, and that each metal has a well marked fatigue limit.

A knowledge of the true value of the fatigue limit of a metal will permit of greater exactitude being attained in the fixing of safe working stresses

Table III.—Torsion Fatigue Limits F_l , and Coefficients C.

Materials and sample marks.		F_l , tons per square inch.	C.
A.A.	Tool steel	7.03	1.91
F. 5108.	"Era" manganese steel	10.40	1.96
F. 5109.	"Era" manganese steel	11.10	2.42
F. 5113.	Manganese steel, 14 per cent. Mn	10.25	1.026
F. 5114.	Manganese steel, 10.5 per cent. Mn	9.60	1.354
F. 5117.	Chrome nickel steel, 12.6 per cent. Ni ...	12.60	0.504
F.A.R.	Farnley iron	6.00	0.615
CI.	Cast iron	3.98	1.61
NI.	Pure rolled nickel	6.22	1.42 to 2.24
CU.	Commercial copper	5.50	0.44
P.B.	Phosphor bronze	7.82	0.72
AL.	Rollled aluminium	2.16	0.127
MA.	Rollled magnalium	4.22	0.077



than is possible when only the ultimate tenacity is known. A knowledge of the relationship between the intensities of fatigue stresses and the numbers of their repetitions will also prove of value in enquiries into the cause of fractures. It has already been of great service in the determination of the stresses which caused a number of copper steam pipes of marine engines to fracture, and may be used for a similar purpose with regard to failures of crankshafts and railway axles.

In conclusion, I beg to thank Prof. Dalby, Prof. A. J. Margetson, and Mr. H. A. Jones for their valuable assistance, and those metallurgists who kindly supplied some of the samples, and especially do I wish to thank the Committee of the Manchester Steam Users' Association for their encouragement of the present and future researches.